MGF 1107

Mathematics of Finance
Handouts

Assistance from the Math Center is permitted.

Tallahassee Community College
MGF 1107
Review of Percent

Calculating Percent Increase or Percent Decrease

1. If 50 is increased to 65, what is the percent increase?

2. If 250 is decreased to 100, what is the percent decrease?

3. If 25 is decreased to 10, what is the percent decrease?

4. If 500 is increased to 600, what is the percent increase?

5. If 36 is increased by 25% of itself, what is the result?

6. If 80 is decreased by 15% of itself, what is the result?

7. If 90 is decreased by 60% of itself, what is the result?

8. If 180 is increased by 5% of itself, what is the result?

Solving for one variable of the sentence $a\%$ of $b = c$.

1. What is 19% of 70?

2. 35 is 40% of what number?

3. 80 is what percent of 120?

4. What is 125% of 44?

5. 150 is what percent of 60?

6. 32 is 80% of what number?

Identifying equivalent decimals, percents, and fractions.

1. 0.017

   (a) $\frac{17}{1000}\%$ (b) 0.00017% (c) $\frac{17}{1000}$ (d) $\frac{17}{100}$

2. 0.026

   (a) 2.6% (b) 0.026% (c) 26% (d) 0.00026%

3. 365%

   (a) 0.365 (b) $\frac{365}{100}\%$ (c) 3.65 (d) $36,500.00$

4. $\frac{35}{20}$

   (a) 1.75% (b) 1.75 (c) 0.0175 (d) 0.0175%

5. $\frac{1}{5}\%$

   (a) $\frac{1}{5}$ (b) $\frac{1}{500}$ (c) 0.20 (d) 0.02

6. $\frac{1}{4}\%$

   (a) 0.25 (b) 2.50 (c) 0.0025 (d) 25

7. $\frac{91}{350}$

   (a) 0.026 (b) 2.6 (c) 26% (d) 0.26%

8. 490%

   (a) 0.490 (b) 4.90 (c) 490.0 (d) 4900.0

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Calculator Practice

Perform the following calculations using your calculator. Copy the entire display of your calculator. The number of digits displayed may vary.

1. $-(-6-8)-(-4-(-2))$

2. $7 \cdot 15^2$

3. $5 \cdot 9 - 8$

4. $4 \cdot (-5)^2$

5. $(4 \cdot (-5))^2$

6. $\dfrac{8 \cdot -6}{2+9}$

7. $\dfrac{\frac{5}{3}}{6}$

8. $\sqrt{3.2}$

9. $(1.05)^{20}$

10. $200(1.02)^{-36}$

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Calculator Practice

Perform the following calculations using your calculator. Round off to the nearest hundredth (as in money to the nearest cent).

1. $200\left[1+(.07)\cdot\frac{5}{6}\right]$

2. $825\,(1+.09)^{15}$

3. $\dfrac{300\left[\left(1+\frac{.12}{4}\right)^{20}-1\right]}{\frac{.12}{4}}$

4. $\dfrac{25\left[\left(1+\frac{.16}{2}\right)^{36}-1\right]}{\frac{.16}{2}}$

5. $\dfrac{820\left[1-\left(1+\frac{.084}{12}\right)^{-360}\right]}{\frac{.084}{12}}$

6. $300\left[1+(.07)\cdot\dfrac{3}{4}\right]$

7. $675(1+.08)^{12}$

8. $\dfrac{100\left[\left(1+\frac{.08}{4}\right)^{16}-1\right]}{\frac{.08}{4}}$

9. $\dfrac{50\left[\left(1+\frac{.18}{2}\right)^{48}-1\right]}{\frac{.18}{2}}$

10. $\dfrac{730\left[1-\left(1+\frac{.072}{12}\right)^{-360}\right]}{\frac{.072}{12}}$

11. $250[1+(0.038)(5)]$

12. $1000\left[1+(0.041)\left(\frac{2}{3}\right)\right]$

13. $7500\left[1+(0.125)\left(\frac{1}{2}\right)\right]$

14. $10,000\left[1+(0.18)\left(\frac{30}{360}\right)\right]$

15. $500(1+0.032)^{12}$

16. $1000\left(1+\frac{.06}{2}\right)^{10}$

17. $750\left(1+\frac{.125}{12}\right)^{36}$

18. $\frac{50\left[(1+.03)^{12}-1\right]}{.03}$

19. $\frac{250\left[\left(1+\frac{.09}{2}\right)^{10}-1\right]}{\frac{.09}{2}}$

20. $\frac{100\left[\left(1+\frac{.18}{12}\right)^{36}-1\right]}{\frac{.18}{12}}$

21. $\frac{750\left[1-(1+.05)^{-10}\right]}{.05}$

22. $\frac{1000\left[1-\left(1+\frac{.062}{12}\right)^{-180}\right]}{\frac{.062}{12}}$

23. $\frac{250\left[1-\left(1+\frac{.084}{12}\right)^{-48}\right]}{\frac{.084}{12}}$

24. $\frac{925\left[1-\left(1+\frac{.072}{12}\right)^{-360}\right]}{\frac{.072}{12}}$

The amount of simple interest earned on the principal P is

$$I = Prt$$

and the amount owed (or in an account) is

$$A = P(1 + rt)$$

where

A is the account balance
P is the principal balance
I is the accrued interest
r is the periodic rate of interest
t is the number of periods interest is accrued

1. Find the interest earned on an investment of \$1200 at 3.5% annual interest for 3 years.

2. Find the interest paid on a \$1500 loan at 4.2% annual interest for 6 months.

3. Find the interest paid on a \$500 investment at 2.8% annual interest for 30 days.

4. Suppose you have \$2500 in your savings account at the end of a certain period of time. You invested \$2000 at a 6.35% simple annual interest rate. How long was your money invested?

5. Suppose you pay back \$520 on a \$500 loan. You had the loan for 60 days. What was your annual simple interest rate?

6. How much should you invest at $2\frac{3}{4}\%$ simple annual interest in order to earn \$100 interest in 3 months?

7. If \$10,000 is invested at 5% simple annual interest, how much will be in the account at the end of 2 years?

8. If \$675 is borrowed for 230 days at 14.5% simple annual interest, how much will be repaid at the end of 230 days?

9. Suppose you pay back a total of \$160 at the end of 120 days on a loan at 18.5% simple annual interest. How much did you borrow?

Computing the Finance Charge on Credit Card Loans

A credit card is a popular way of making purchases or borrowing money. The finance charges on credit cards are usually calculated using the average daily balance method. With the average daily balance method, a balance is determined each day of the billing period for which there is a transaction in the account. The finance charge for the month is found using the simple interest formula with the average daily balance as the principal.

Example

Jim's Mastercard bill has arrived for the month of January. The bill shows an average daily balance of \$2500. The annual rate of interest is 18%. Find the monthly finance charge.

$$I = Prt$$
$$P = \$2500$$
$$r = 18\% = 0.18$$
$$t = \frac{1}{12}$$

$$I = 2500(0.18)\left(\frac{1}{12}\right) = \$37.50$$

Example

Mary's Visa bill has also arrived for the month of January. The bill shows an average daily balance of \$2500. The monthly rate of interest is 1.8%. Find the monthly finance charge.

$$P = \$2500$$
$$r = 1.8\% = 0.018$$
$$t = 1$$

$$I = 2500(.018)(1) = \$45.00$$

Find the monthly finance charge on a balance of \$4000 for each of the following interest rates:

1. 18.5% annually
2. 23.25% annually
3. 12.5% annually
4. 1.85% monthly
5. 2.25% monthly
6. 1.5% monthly

When interest is deposited to the account periodically, the balance is increased and the new balance earns interest. The formula for computing the balance in an account with the interest compounded periodically is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where

A is the account balance
P is the principal or beginning balance
r is the annual interest rate
n is the number of compounding periods per year
t is the number of years

The effective yield is the equivalent rate that one would earn as though the money were earning simple interest. The effective yield is given by

$$y = \left(1 + \frac{r}{n}\right)^{n} - 1$$

In each of the following problems, round your answers to the nearest penny.

1. You have \$100 to invest. How much money will you have at the end of two years at 5% if interest is compounded

   (a) annually?

   (b) semiannually?

   (c) quarterly?

   (d) monthly?

   (e) daily?

   How much interest was earned in each of the cases?

2. Find the future value and the interest earned on an investment of \$4000 invested for 20 years at 5.2% compounded annually.

3. Find the future value and the interest earned on an investment of \$300 invested for 12 years at 2.5% compounded semiannually.

4. Find the future value and the interest earned on an investment of \$2500 invested for 8 years at 6.32% compounded quarterly.

5. Find the future value and the interest earned on an investment of \$100,000 invested for 6 years at 8.5% compounded daily.

6. How much money should you invest today at 4.8% interest compounded monthly in order to have \$10,000 saved for a nice vacation in 5 years?

7. Joe and Joan have a new grandson. How much money should they invest now so that he will have \$50,000 for his college education in 18 years? The money is invested at 8.5% compounded annually.

8. How much should be invested today at 5.8% compounded semiannually to have $15,000 in 10 years?

9. Donna wants $30,000 saved in 3 years to make a down payment on a house. How much money should she invest now at 5.2% compounded quarterly in order to meet her goal?

10. How much should be invested now at 6.5% compounded monthly to have $40,000 in 15 years?

11. Find the effective yield of an investment that earns 12.5% compounded monthly.

12. Find the effective yield on an investment that pays 5.2% compounded quarterly.

13. Which is the better investment: 5.1% compounded daily or 5.15% compounded monthly? (Compare the effective yields.)

1. How much should you invest now in order to have \$10,000 in 8 years if your investment can earn 6.5% compounded quarterly?

2. How much interest do you earn in problem 1?

3. If you invest \$6,000 now, how long will it take for your investment to be worth \$10,000 if it earns 7.4% compounded monthly?

4. If the inflation rate is 5% compounded annually, how long will it take for prices to double?

5. If your retirement pay is increasing at a rate of 3% compounded annually, how long will it take for your retirement pay to increase from \$25,000 to \$40,000?

6. How much can you borrow for 6 years if you only want to repay $3000 and the interest rate is 9.6% compounded weekly?

7. How much interest did you pay in problem 6?

8. You invest $1700 now and want the investment to be worth $5000 in 12 years. What interest rate (compounded annually) do you need to earn? Round to the nearest hundredth of a percent.

9. You borrow $2500 for 4 years and only want to repay $3000. What rate of interest (compounded monthly) should you pay? Round to the nearest hundredth of a percent.

10. You purchased a home 10 years ago that cost $84,000. The home is now worth $150,000. What was the rate of appreciation in value of the home (growth rate compounded annually)? Round to the nearest hundredth of a percent.

Definition: A sequence of payments into or out of an interest-bearing account is called an annuity.

Ordinary Annuity:

1. The payments are made at the end of each time period.
2. The frequency of payments must match the frequency of the compounding.
3. The periodic payment is always the same amount of money.

The amount of the annuity is the sum of all the payments plus all accumulated interest. The future value of an ordinary annuity is given by

$$A = \frac{m\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\left(\frac{r}{n}\right)}$$

Where

- A is the future value of the annuity. It is the sum of all the periodic deposits plus the interest earned on those deposits. Sometimes the symbol FV is used.
- m is the periodic payment. Sometimes the symbol pmt is used.
- r is the annual interest rate
- n is the number of compounding periods per year.
- t is the number of years.

Example 1
Find the future value of an ordinary annuity whose quarterly payment is \$1500 at 6.5% compounded quarterly for 5 years.

Example 2
Mrs. Bernath is participating in the DROP program. The Florida Retirement System is depositing \$2200 a month into an annuity that pays 6.5% compounded monthly. She plans to work until December, 2005. At the end of that time, 30 monthly payments will have been made into her account. How much money will be in her account when she says good-bye to all the wonderful people at TCC?

Ordinary Annuity Problems

1. You invest \$150 each month into an annuity that earns a fixed rate of return of 7.5% compounded monthly. In 30 years, how much will you have in your annuity?

2. How much interest did you earn in problem 1?

3. You would like to have \$800,000 when you retire in 35 years. How much should you invest each quarter if you can earn a rate of 6.5% compounded quarterly?

4. How much interest do you earn in problem 3?

5. A company needs \$4,000,000 in 10 years to expand. How much should the company invest each week if the investment can earn a rate of 11% compounded weekly?

6. Each year you invest \$2000 into an annuity that earns a rate of 6.5% compounded annually. How much do you have in the account after investing for 20 years?

7. How much should you invest each month in order to have \$500,000 if your rate of return is 7% compounded monthly and you want to achieve your goal of \$500,000 in 40 years?

8. How much interest do you earn in problem 7?

9. How much should you invest each month if you want to achieve your goal in problem 7 (\$500,000) in only 20 years?

10. If you invest each month the amount that was the answer to problem 9, but only invest for 10 years, how much is your investment worth?

Amortization problems are similar to annuity problems in that both involve a sequence of payments and interest is paid on a balance. The difference is that here we are talking about paying off a balance that is owed. The amount of money that is owed is called the present value. Some common types of amortization problems involve mortgages, auto loans, and credit cards.

The formula used is

$$P = \frac{m\left[1-\left(1+\frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)}$$

Where

- P is the present value or the current balance. Sometimes the symbol PV is used.
- m is the periodic payment. Sometimes the symbol pmt is used.
- r is the annual rate of interest
- n is the number of compounding periods per year
- t is the length of time needed to retire the debt

Problems

1. You want to buy a new car. You can afford payments of \$250 per month and can borrow the money at an interest rate of 5.9% compounded monthly for 5 years. How much are you able to borrow?

2. How much interest do you pay in problem 1?

3. If you want to borrow the money in problem 1 for only 3 years, how much are you able to borrow? (assume the same payment and interest rate)

4. You owe $5000 on student loans at an interest rate of 7% compounded annually. You want to pay the loan off in 8 years. How much should you pay each year?

5. How much interest do you pay in problem 4?

6. You decide to quit using your credit card and want to pay off the balance of $4500 in two years. Your interest rate is 14.9% compounded monthly. How much should you pay each month?

7. You buy a house that costs $120,000. You pay 10% down and finance the balance. Your rate of interest is 7% compounded monthly, and your loan is for 30 years. How much are your monthly payments?

8. How much interest did you pay in problem 7?

9. If your loan in problem 7 is for 15 years, how much are your monthly payments?

10. How much interest do you pay in problem 9?

11. You can afford monthly payments of $800 per month and can borrow the money at an interest rate of 7% compounded monthly for 30 years. How much can you borrow?

12. The Youngs plan to buy a house with a purchase price of $125,000. They have saved enough for a 10% down payment. They plan to borrow the remainder from their bank. They can get a 30 year, 6.635% mortgage. What is their monthly payment? How much interest will they pay if they keep the mortgage for 30 years?

13. If the bank offers the same interest rate for a 15-year mortgage, how much interest would the Youngs save by going with the 15-year loan?

14. The same bank will give the Youngs a 15-year loan at a fixed rate of 6.375%. What is the monthly payment? How much interest will they pay if they keep the mortgage for 15 years? How much will they save if they choose the 15-year mortgage rather than the 30 year mortgage?

15. List one consideration that would determine whether a couple would agree to the 15-year as opposed to a 30-year mortgage.

The following problems are of all the types discussed in this unit. You will need to determine the type of problem it is, which formula should be used, and apply the formula correctly.

1. Find the monthly finance charge on Karen's credit card that has an average daily balance of \$1700. The annual interest rate is 21.8%.

2. Terri borrowed \$250 at 25% simple annual interest for 30 days. How much did she repay? How much of this was interest?

3. Carolyn and Lee are saving for their fortieth anniversary party in 10 years. They decide that \$5000 will be their goal. They make semiannual payments into an annuity that pays 6.2% compounded semiannually. How large are their payments? How much interest do they earn?

4. How much money should Norman invest today at 6.75% compounded annually in order to have \$20,000 in 8 years? How much interest does he earn?

5. Sam borrowed \$500 from a loan shark for 180 days. He repaid the loan shark \$587.50. What simple annual interest rate did the loan shark charge? Express the rate as a percent.

6. Ray has decided to buy a new car. He knows he can afford $425 a month for the payments. A local car dealer will give him a 5 year loan at 5.9% compounded monthly. How much money can Ray borrow now? How much will he pay back over the 5 years? How much total interest will he pay?

7. Lula invested $3200 at 5.2% interest compounded quarterly. How much money will be in the account at the end of 7 years? How much of this is interest?

8. How long does it take $750 to double if it is invested at 7% compounded annually? (Round answer to the next full year)

9. Find the effective yield of money invested at 7.13% compounded daily. Round your answer to the nearest hundredth of a percent.

10. Selena wants to set up an annuity for her future retirement. She decides to contribute $1800 a year into an account that pays 5.80% compounded annually. How much money will Selena have in 35 years? How much will she contribute to the annuity? How much interest will she earn?

11. Mr. and Mrs. Sharp are buying a new home. The home they are buying costs $200,000 and they are making a 20% down payment. They have obtained a 30 year mortgage for the balance at a fixed rate of 7.30% compounded monthly. Find

    (a) the amount of the down payment

    (b) the amount they must borrow

    (c) the monthly payment

    (d) the total repaid, if they keeep the mortgage for the entire 30 years

    (e) the total amount of interest paid

    (f) How much of the first monthly payment is interest?

12. You decide to deposit $875 per quarter into an account earning 8% compounded quarterly. What is the value of your ordinary annuity at the end of the 20 years? How much interest did you earn?

13. Assume the rate of inflation is 5% per year compounded annually. What would a house be worth today if it will be worth $200,000 nine years from now? How long will it take for prices to double?

14. Use effective yields to determine which of the following is the better investment:

    7.7% compounded quarterly or 7.6% compounded weekly

15. You borrow \$5700 for 10 months at an annual simple interest rate of 6.5%. How much interest are you charged? How much do you repay at the end of the 10 months?

16. If tuition increases by 7% compounded annually for the next 12 years, what will a credit hour that costs \$40 now cost in 12 years?

17. You buy a stock for \$125 and sell it 7 years later for \$306. What rate of interest compounded annually do you earn?

18. You buy a house that costs \$140,000 by paying 20% down and financing the balance over 30 years at 7% interest compounded monthly.

    (a) What are your monthly payments?

    (b) How much total interest do you pay during the 30 years?

19. You invest \$5000 for 80 weeks and receive back \$5300. What simple annual interest rate did you earn?

20. If you want to have \$10,000 in seven years, how much should you deposit each month into an account paying 8.5% compounded monthly?

21. You invest \$2000 at 9% compounded semi-annually. How long should the money stay invested to be worth \$7000?

22. You can afford car payments of \$200 per month. Assume that the interest rate is 5.9% compounded monthly and you finance the car for 4 years. What is the amount of money that you are able to borrow?

Solutions

Page 1

1. 30%
2. 60%
3. 60%
4. 20%
5. 45
6. 68
7. 36
8. 189

1. 13.3
2. 87.5
3. $66\frac{2}{3}\%$
4. 55
5. 250%
6. 40

Page 2

1. C
2. A
3. C
4. B
5. B
6. C
7. C
8. B

Page 3

1. 16
2. 1575
3. 37
4. 100
5. 400
6. $-4.\overline{36}$
7. $\frac{5}{18} = 0.2\overline{7}$
8. 1.788854382
9. 2.653297705
10. 98.04463007

Page 4

1. 211.67
2. 3005.05
3. 8061.11
4. 4677.55
5. 107,634.48
6. 315.75
7. 1699.76
8. 1863.93
9. 34,214.02
10. 107,544.59
11. 297.50
12. 1027.33
13. 7968.75
14. 10150.00
15. 729.67
16. 1343.92
17. 1089.13
18. 709.60
19. 3072.05
20. 4727.60
21. 5791.30
22. 117,000.10
23. 10,162.14
24. 136,272.26

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1. \$126
2. \$31.50
3. \$1.17
4. 3.9 years
5. 24%
6. \$14,545.46
7. \$11,000
8. \$737.53
9. \$150.71

Page 8

1. \$61.67
2. \$77.50
3. \$41.67
4. \$74.00
5. \$90.00
6. \$60.00

Page 9

1. (a) 110.25, $i = 10.25$
   (b) 110.38, $i = 10.38$
   (c) 110.45, $i = 10.45$
   (d) 110.49, $i = 10.49$
   (e) 110.52, $i = 10.52$
2. \$11,024.90, $i = 7,024.90$
3. \$404.21, $i = 104.21$
4. \$4128.60, $i = 1628.60$
5. \$166,519.09, $i = 66,519.09$
6. \$7870.05
7. \$11,514.23
8. \$8468.07
9. \$25,692.59
10. \$15,127.45
11. 13.24%
12. 5.30%
13. 5.15% compounded monthly

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1. 5970.11
2. 4029.89
3. Approx. 6.924 years, or 6 years, 11 months, and 3 days. (The interest would not be credited until the end of the seventh year.)
4. 14.21 years
5. 15.9 years
6. 1687.32
7. 1312.68
8. 9.41%
9. 4.57%
10. 5.97%

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Example 1 \$35,115.67
Example 2 \$71,455.68

1. 202,116.81
2. 148,116.81
3. 1520.18
4. 587,174.80
5. 4229.34
6. 77,650.62
7. 190.49
8. 408,564.80
9. 959.83
10. 166,131.99

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1. 12,962.54
2. 2037.46
3. 8230.01
4. 837.34
5. 1698.72
6. 217.98
7. 718.53
8. 150,670.80
9. 970.73
10. 66,731.40
11. 120,246.05
12. $m = 721.09$
    $i = 147,092.40$
13. 81,687.60
14. $m = 972.28$
    $i = 62,510.40$

1. 30.88
2. $A = 255.21$
   $i = 5.21$
3. $m = 184.19$
   $i = 1316.20$
4. $P = 11,860.06$
   $i = 8139.94$
5. 35%
6. $P = 22,036.33$
   $i = 3463.67$
7. $A = 4594.25$
   $i = 1394.25$
8. 10.25 years
9. 7.39%
10. $A = 192,241.95$
    63,000 contributed
    $i = 129,241.95$
11. (a) 40,000
    (b) 160,000
    (c) 1096.91
    (d) 394,887.60
    (e) 234,887.60
    (f) 973.33
12. $A = 169,550.46$
    $i = 99,550.46$
13. $P = 128,921.78$
    14.21 years
14. 7.7% quarterly
15. $i = 308.75$
    6008.75
16. 90.09
17. 13.64%
18. $m = 745.14$
    $i = 156,250.40$
19. 3.9%
20. 87.53
21. 14.23 or
    $14\frac{1}{2}$ years
22. 8532.71